Journal of Statistical Software

MMMMMM YYYY, Volume VV, Issue II. http://www.jstatsoft.org/

Dip Test Distributions, P-values, and other Explorations

Martin Mächler
ETH Zurich

Abstract

...
...

Keywords: MPFR, Abitrary Precision, Multiple Precision Floating-Point, R.

```
 [1] "/sfs/u/staff/maechler/R/BioCore/library"
 [2] "/sfs/u/staff/maechler/R/library/64-linux-MM-only"
 [3] "/sfs/u/staff/maechler/R/library/64-linux"
 [4] "/sfs/u/staff/maechler/R/library/Linux"
 [5] "/sfs/u/staff/sfs/R/library/64-linux"
 [6] "/sfs/u/staff/sfs/R/library/Linux"
 [7] "/sfs/s/linux/rhel3_amd64/app/R/R_local/library-Recommended"
 [8] "/sfs/s/linux/rhel3_amd64/app/R/Bioconductor/library_2.8"
 [9] "/sfs/s/linux/rhel3_amd64/app/R/R_local/library"
[10] "/sfs/u/staff/maechler/R/D/r-patched/F13-64-inst/library"
```

1. Introduction

FIXME: Need notation

D_n :=`dip( runif(n) )`;

but more generally,

$$D_n(F) := D(X_1, X_2, \dots, X_n), \quad \texttt{where } X_i \text{ i.i.d. } , X_i \sim F. \tag{1}$$

Hartigan and Hartigan (1985) in their "seminal" paper on the dip statistic D_n already proved that $\sqrt{n}\, D_n$ converges in distribution, i.e.,

$$\lim_{n \to \infty} \sqrt{n}\, D_n \overset{\mathcal{D}}{=} D_\infty. \tag{2}$$

A considerable part of this paper is devoted to explore the distribution of D_∞.

2. History of the diptest R package

Hartigan (1985) published an implementation in Fortran of a concrete algorithm, where the code was also made available on Statlib[1]

- MM started in 1994, with S-plus code interfacing to Hartigan's Fortran

- several important bug fixes; last one Oct./Nov. 2003

However, the Fortran code file http://lib.stat.cmu.edu/apstat/217, was last changed Thu 04 Aug 2005 03:43:28 PM CEST

We have some results of the dip.dist of *before* the bug fix; notably the "dip of the dip" probabilities have changed considerably!!

- see rcslog of ../../src/dip.c

3. 21st Century Improvement of Hartigan[2]'s Table

((

Use listing package (or so to more or less "cut & paste" the nice code in `../../stuff/new-simul.Rout-1e6`

))

4. The Dip in the Dip's Distribution

We have found empirically that the dip distribution itself starts with a "dip". Specifically, the minimal possible value of D_n is $\frac{1}{2n}$ *and* the probability of reaching that value,

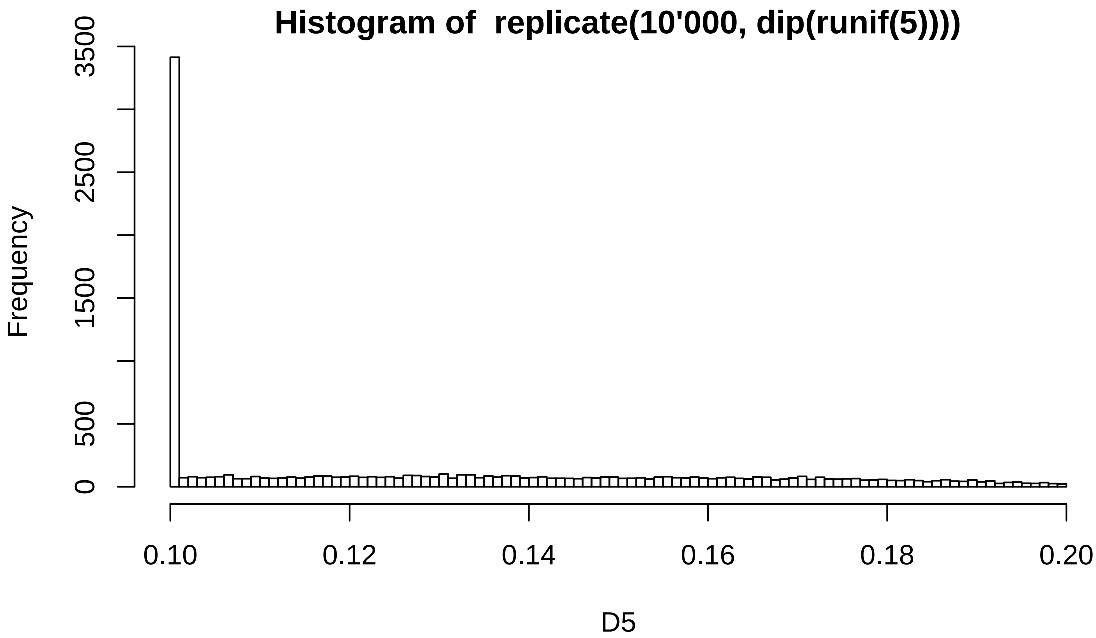
$$\mathrm{P}\left[D_n = \frac{1}{2n}\right], \tag{3}$$

is large for small n.

E.g., consider an approximation of the dip distribution for $n = 5$,

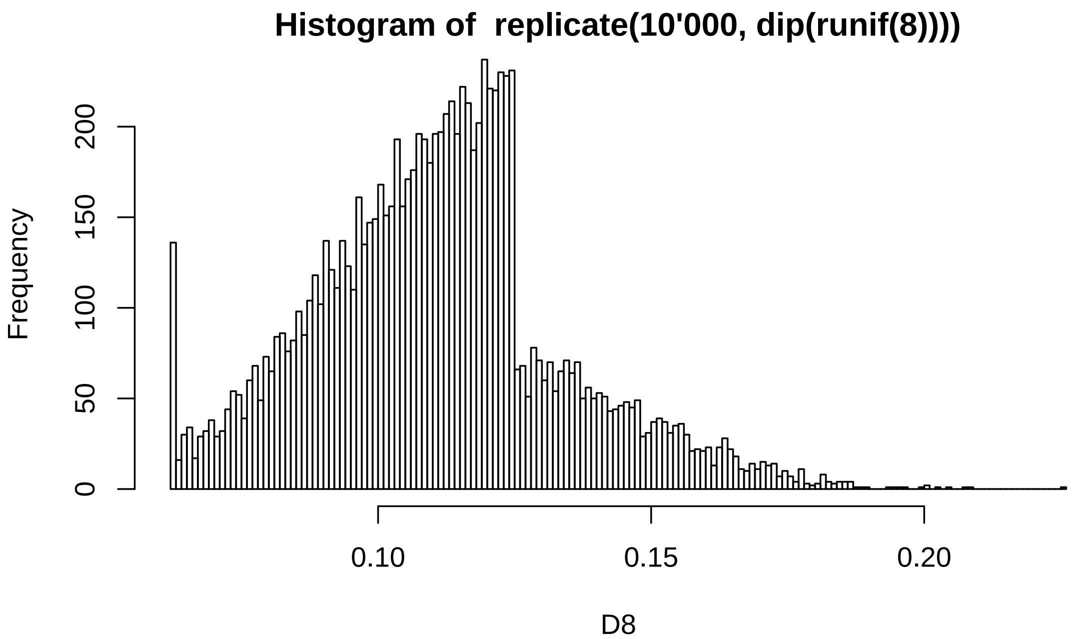
```
R> require("diptest") # after installing it ..
R> D5 <- replicate(10000, dip(runif(5)))
R> hist(D5, breaks=128, main = "Histogram of  replicate(10'000, dip(runif(5))))")
```

[1]Statlib is now a website, of course, http://lib.stat.cmu.edu/, but then was *the* preferred way for distributing algorithm for statistical computing, available years before the existence of the WWW, and entailing e-mail and (anonymous) FTP



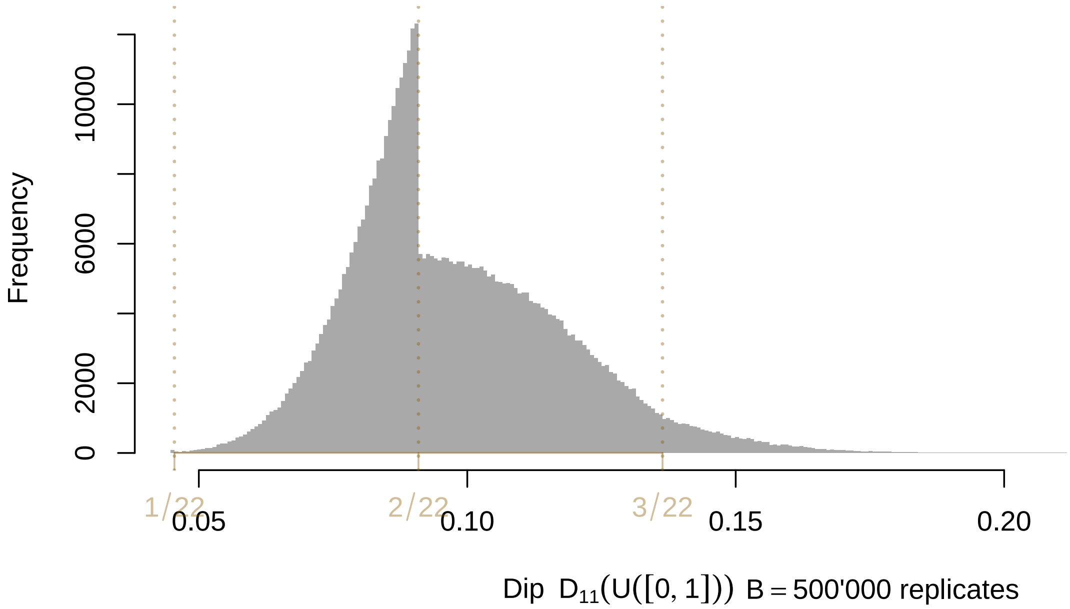
which looks as if there was a bug in the software — but that look is misleading! Note how the phenomenon is still visible for $n = 8$,

```
R> D8 <- replicate(10000, dip(runif(8)))
R> hist(D8, breaks=128, main = "Histogram of  replicate(10'000, dip(runif(8))))")
```



Note that there is another phenomenon, in addition to the point mass at $1/(2n)$, particularly visible, if we use *many* replicates,

```
R>  set.seed(11)
R>  n <- 11
R>  B.s11 <- 500000
R>  D11 <- replicate(B.s11, dip(runif(n)))
```



FIXME:
use '../../stuff/sim-minProb.R'
and '../../stuff/minProb-anal.R'

Further, it can be seen that the *maximal* dip statistic is $\frac{1}{4} = 0.25$ and this upper bound can be reached simply (for even n) using the the data $(0, 0, \ldots, 0,\ 1, 1, \ldots, 1)$, a bi-point mass with equal mass at both points.

5. P-values for the Dip Test

Note that it is not obvious how to compute P-values for "the dip test", as that means determining the distribution of the test statistic, i.e., D_n under the null hypothesis, but a natural null, $H_o : F \in \{F \text{cadlag} \mid f := \frac{d}{dx} F is unimodal\}$ is too large. Hartigans'(1985) argued for using the uniform $U[0, 1]$ i.e., $F'(x) = f(x) = \mathbf{1}_{[0,1]}(x) = [0 \le x \le 1]$ (Iverson bracket) instead, even though they showed that it is not quite the "least favorable" one. Following Hartigans', we will define the P-value of an observed d_n as

$$P_{d_n} := \mathrm{P}\left[D_n \ge d_n\right] := \mathrm{P}\left[\mathrm{dip}(U_1, \ldots, U_n) \ge d_n\right], \quad \text{where } U_i \sim U[0, 1], \quad \text{i.i.d.} \tag{4}$$

5.1. Interpolating the Dip Table

Because of the asymptotic distribution, $\lim_{n\to\infty} \sqrt{n}\, D_n \stackrel{\mathcal{D}}{=} D_\infty$, it is makes sense to consider the "$\sqrt{n} D_n$"–scale, even for finite n values:

```
R> data(qDiptab)
R> dnqd <- dimnames(qDiptab)
R> (nn. <- as.integer(dnqd[["n"]]))
```

```
 [1]    4    5    6    7    8    9   10   15   20   30   50  100  200  500
[15] 1000 2000 5000
```

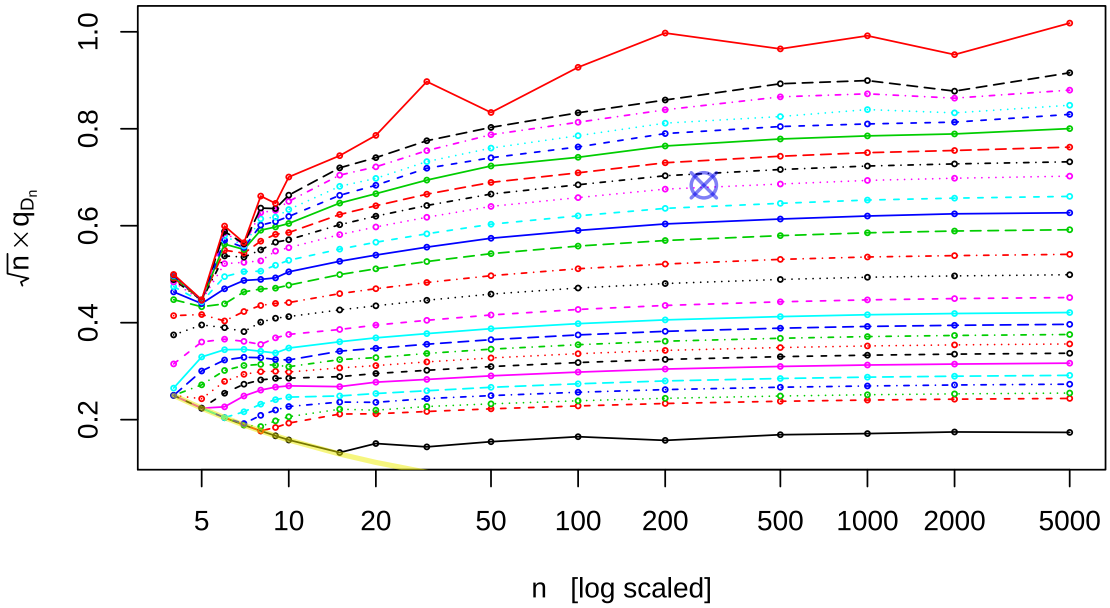
```
R> matplot(nn., qDiptab*sqrt(nn.), type ="o", pch=1, cex = 0.4,
          log="x", xlab="n   [log scaled]",
```

```
        ylab = expression(sqrt(n) %*% q[D[n]]))
R> ## Note that  1/2n  is the first possible value (with finite mass),,
R> ## clearly visible for (very) small n:
R> lines(nn., sqrt(nn.)/(2*nn.), col=adjustcolor("yellow2",0.5), lwd=3)
R> P.p <- as.numeric(print(noquote(dnqd[["Pr"]])))
```

```
 [1] 0       0.01    0.02    0.05    0.1     0.2     0.3     0.4
 [9] 0.5     0.6     0.7     0.8     0.9     0.95    0.98    0.99
[17] 0.995   0.998   0.999   0.9995  0.9998  0.9999  0.99995 0.99998
[25] 0.99999 1
```

```
R> ## Now look at one well known data set:
R> D <- dip(x <- faithful$waiting)
R> n <- length(x)
R> points(n, sqrt(n)*D, pch=13, cex=2, col= adjustcolor("blue2",.5), lwd=2)
R> ## a simulated (approximate) P-value for D  is
R> mean(D <= replicate(10000, dip(runif(n)))) ## ~ 0.002
```

```
[1] 0.0015
```



but we can use our table to compute a deterministic (but still approximate, as the table is from simulation too) P-value:

```
R> ## We are in this interval:
R> n0 <- nn.[i.n <- findInterval(n, nn.)]
R> n1 <- nn.[i.n +1] ; c(n0, n1)
```

```
[1] 200 500
```

```
R> f.n <- (n - n0)/(n1 - n0)# in [0, 1]
R> ## Now "find" y-interval:
R> y.0 <- sqrt(n0)* qDiptab[i.n  ,]
R> y.1 <- sqrt(n1)* qDiptab[i.n+1,]
R> (Pval <- 1 - approx(y.0 + f.n*(y.1 - y.0),
                    P.p,
                    xout = sqrt(n) * D)[["y"]])
```

```
[1] 0.001809527
```

```
R> ## 0.018095
```

5.2. Asymptotic Dip Distribution

We have conducted extensive simulations in order to explore the limit distribution of D_∞, i.e., the limit of $\sqrt{n}\, D_n$, (2).

Our current R code is in ' `../../stuff/asymp-distrib.R` ' but the simulation results (7 Megabytes for each n) cannot be assumed to be part of the package, nor maybe even to be simply accessible via the internet.

6. Less Conservative Dip Testing

7. Session Info

```
R> toLatex(sessionInfo())
```

- R version 2.13.0 Patched (2011-05-17 r55946), `x86_64-unknown-linux-gnu`
- Locale: `LC_CTYPE=de_CH.UTF-8`, `LC_NUMERIC=C`, `LC_TIME=en_US.UTF-8`, `LC_COLLATE=de_CH.UTF-8`, `LC_MONETARY=C`, `LC_MESSAGES=C`, `LC_PAPER=de_CH.UTF-8`, `LC_NAME=C`, `LC_ADDRESS=C`, `LC_TELEPHONE=C`, `LC_MEASUREMENT=de_CH.UTF-8`, `LC_IDENTIFICATION=C`
- Base packages: base, datasets, graphics, grDevices, methods, stats, tools, utils
- Other packages: diptest 0.50-0, fortunes 1.4-1, sfsmisc 1.0-14

References

Hartigan JA, Hartigan PM (1985). "The Dip Test of Unimodality." *Annals of Statistics*, **13**, 70–84.

Hartigan PM (1985). "Computation of the Dip Statistic to Test for Unimodality." *Applied Statistics*, **34**, 320–325.

Affiliation:

Martin Mächler
Seminar für Statistik, HG G 16
ETH Zurich
8092 Zurich, Switzerland
E-mail: maechler@stat.math.ethz.ch
URL: http://stat.ethz.ch/people/maechler

Journal of Statistical Software http://www.jstatsoft.org/
published by the American Statistical Association http://www.amstat.org/

Volume VV, Issue II *Submitted:* yyyy-mm-dd
MMMMMM YYYY *Accepted:* yyyy-mm-dd